Incorporating Critical and Creative Thinking in a Transitions Course

Abstract. In this note, I discuss my experiences explicitly incorporating principles of critical and creative thinking in a transitions course which serves mathematics, mathematics education, and statistics majors. I describe several specific assignments and classroom tasks designed to enhance critical and creative thinking. I also discuss some evidence from survey instruments.

1. Language and Practice of Critical and Creative Thinking

Transitions or introduction-to-proof courses — designed to bridge the gap between computationally-focused courses up through calculus and rigorous proof-based courses — are a notorious problem of pedagogical design and practice. How do we help the beginning student see that there is more to mathematics than being very good at certain algorithms?

In this note, I discuss my experiences explicitly incorporating principles of critical and creative thinking (CCT) in a transitions course which serves mathematics, mathematics education, and statistics majors. This work was part of a campus-wide quality-enhancement initiative (think.dasa.ncsu.edu) targeted toward first- and second-year courses. My course was the only mathematics course selected for the program. Actual implementation in the classroom was supported by faculty-peer mentoring throughout the two semesters of the program, and an intensive two-day workshop in May prior to the August start of the implementation.

The framework used for CCT is a set of standards (the "Intellectual Standards") adapted from [4]. Though there are definitions for each Standard, I prefer to give students illustrating questions for each Standard, as given in Table 1. These questions are written in the first person, as questions the student might ask themself while working on a class assignment.

As a cross-campus initiative, one of the most important aspects of TH!NK is using a common language of critical and creative thinking; that is, consistent usage of the thirteen named Standards. The motivation for this is twofold: first, for students who have taken or are concurrently taking another TH!NK course, the lessons in critical and creative thinking are reinforced; second (and more importantly) this consistency across disciplines communicates to students that the specialized ways we think in mathematics are outgrowths of general principles of critical and creative thought processes that also apply in other academic disciplines and outside of academic contexts as well. (An ancillary benefit is to remind participating faculty, as well, that their disciplines are not so siloed.)

One can — and in the cross-disciplinary training/planning sessions many did — quibble with the both the particular labels and the division of critical and creative thinking into the Intellectual Standards. For example, the distinction between *accuracy* and *precision* as given by Paul and Elder is not entirely firm, and is not

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Standards of Critical Thinking	
clarity	*What does this mean?*
accuracy	*How could we check that?*
precision	*Could I be more specific?*
relevance	*How does that help us with the issue?*
depth	*What factors make this a difficult problem?*
	What are some of the complexities of this question?
breadth	*Do we need to look at this from another perspective?*
logic	*Does this follow from the evidence?*
	Does all this make sense together?
significance	*Is this the central idea to focus on?*
	Is this an important problem to consider?
fairness	*Do I have any vested interest in this issue?*
Standards of Creative Thinking	
originality	*Is this a new idea?*
adaptability	*How do I need to adjust my thinking to new contexts?*
appropriateness	*Do the properties of my solution fit the constraints of the problem?*
contribution to the domain	*What is the value of this idea to the discipline?*

TABLE 1. The Intellectual Standards and illustrating questions.

the standard distinction between the way these words are used in laboratory science. Nevertheless, having an agreed-upon label for the suite of questions associated to each Intellectual Standard is convenient in practice. In the context of a transitions course, this issue (which at least one of your students will raise!) also offers an opportunity for an in-class discussion of the difference between *the label* and *the definition* that label attaches to.

In addition to the principles of critical and creative thinking, as expressed in the thirteen Intellectual Standards, there are several important Practices of critical and creative thinking. The most relevant Practices for my course are: *reflection*, *risk-taking*, and *revision*.

2. Implementation

In this section I will describe the curricular structure by which I hope to teach my students to engage in CCT in the context of proof writing, and mathematics more generally.

2.1. Revise, revise, revise! The course is graded using a variant of specifications grading [3]. I assign around 120 proof problems, from which students must compile an end-of-semester proof portfolio that meets certain binary criteria such *submit a proof by induction*, *submit a proof of a calculus fact*, and *submit a proof of an open-ended problem*. Following a suggestion by Robert Talbert [5], each proof receives one of three possible marks: *satisfactory* proofs are complete and correct, *unsatisfactory* proofs are essentially flawed, and *progressing* proofs have some correctable flaws and can be resubmitted. Only satisfactory proofs are eligible for inclusion in the final portfolio, i.e. count toward the final grade.

This system combines the specifications systems goal of only rewarding complete work (a proof must be essentially perfect to count), while encouraging students to engage thoughtfully with feedback — they must revise a progressing proof in order for it to count at all toward the final grade. Some students require several attempts to get a proof into *satisfactory* shape; I allow them unlimited attempts, constrained only by the hard deadline of the end of the semester.

As noted below, the two creative assignments require an initial submission followed by feedback and revision.

2.2. Reflective practice. Nearly every component of the course has a short reflective component. Weekly homework assignments are accompanied by a reflection journal, which contains several different levels of reflective questions. Some questions ask students to reflect on their work product:

- In proof 4, what facts about even and odd numbers did you use?

There are also some questions which ask students to reflect on their process:

- Much of the work of writing a proof is chasing down dead ends. What are two things you tried on this assignment that did not work? For each, reflect on why you thought the approach would work, and why it did not work.

Other questions ask them to reflect on the framing of the problem:

- Problems 3 - 5 are about fractions; why didn't I allow you to *use* fractions in these proofs?

Some questions go beyond reflection to ask students to evaluate the framing of the problem:

- In proof 1, we use the notation 2^n to represent a number. We have also used the notation 2^A to represent the powerset of a given set (the set of subsets of A). So we are using the same notation for two entirely different things. What do you think about this? Might it cause confusion? Is this something we should worry about?

The creative assignments described below have reflective components for each stage as well, including the following, which reinforces both the Intellectual Standards and the idea of self-evaluation:

- Among the Intellectual Standards of Critical and Creative Thinking, pick three you might be appropriate to evaluate your work on this assignment.

2.3. Risk-taking. One frequent stumbling block in proof-writing is for the student to be able to articulate and manipulate statements they know to be false. This skill, after all, is required to read and write a *reductio ad absurdum.* In my experience, students' hestitancy around false statements is one fact that makes material implication so difficult to learn. I also have the goal of my classroom being a welcoming and safe environment where students are free to make mistakes. The following in-class partner task starts to address both issues, on the first day of class.

Each student is given a slip of paper, on which is written a plausible-sounding but false statement. The students receive the following instructions:

(1) Introduce yourself to your partner.
(2) Read your sentence to yourself.
(3) Read your sentence aloud to your partner.
(4) Decide together whether your sentences are statements.

(5) Quietly write down:
  - what it would mean for your sentence to be false
  - what it would mean for your partner's sentence to be false

(6) Read your sentence aloud to your partner (again).

Once this task is complete, the students receive further instructions:

(1) Congratulations! Your sentence is false.

(2) Pick an 'A' partner and a 'B' partner and perform this dialog:
    A: [read your sentence aloud]
    B: That is false because [give your reason].
    A: Thank you for finding my mistake.

(3) Repeat with roles reversed.

This activity leverages mild role-play [6] to introduce students to the important ideas firstly that it is *socially* acceptable to state something incorrect within the context of the classroom, and secondly that it is often *mathematically* necessary to deal in false statements.

2.4. Explicit discussion of the Standards. Early in the semester, I introduce the thirteen Standards via the following discussion activity. I ask the students to sort each Intellectual Standard according to whether it might be relevant to the *formatting* of a proof, the *style* of a proof, the *content* of a proof, or the *process* of writing a proof. I record results of the resulting (lively!) class discussion, and then reprise the exercise near the end of the semester after the students have gained more experience in writing proofs.

2.5. Using the Standards in grading and feedback. To some extent, the specifications grading system separates marking of assignments from per-assignment grades. This is freeing both for the student and for me. Instead of thinking in terms of errors and how many points they are worth, I have more time and energy to spend making remarks like *What does "vice versa" mean in this context? Be precise*. Sometimes my comment is simply the name of the relevant Standard (most often *Relevance?* and *Clarity!*).

My peer feedback forms, implemented using Google Drive, ask the students to comment expressly on the Standards as expressed in their peers work. In the induction analogy assignment described below, for example, students are asked to evaluate the *clarity*, *accuracy*, *relevance*, *depth*, *originality*, and *appropriateness* of each others work.

2.6. Creative assignments. One important lesson of a transitions course is that mathematics is at its core a creative endeavor. Some of the frustrations students experience are due to the particular characteristics of creative thinking. I address this head-on by making two major creative assignments during the semester.

The induction analogy assignment, based on one I first heard described in a talk at the Centennial MathFest [2], is a multipart assignment which requires students to write their own analogies for induction, as follows:

> Part 1: the basic analogy. Select a word that starts with the same letter as your family name. Build an analogy for induction using that word. You must identify:
> - the base case $P(1)$
> - the inductive hypothesis $P(n)$

- the inductive step $P(n) \Rightarrow P(n+1)$
- the conclusion $\forall n, P(n)$

Your analogy must include a visual representation of some kind.

Part 2: peer feedback. Read your peer's analogy, and provide them feedback with the following questions in mind: Does the analogy feature each of the four parts of a proper proof by induction? Is the writeup true to the nonmathematical subject of the analogy? Suggest a way to make the analogy more forceful.

Part 3: revision and extension. Revise your analogy according to the feedback you receive from your peers and me. Extend the analogy to explain strong induction and the Well-Ordering Principle.

Some examples of words selected by students, and their justifications (provided here in my own words):

phalanx: the row of phalangites interlocked their shields, each protecting the soldier to his left; the rightmost phalangite is $P(1)$.

hammer: When you get a good hammering rhythm going, the rebound from each strike sets up the next blow. This is the inductive step.

Further details of the induction analogy assignment can be found in the preprint [1].

The second major creative assignment is similar in nature, though smaller in scope. Students select an equivalence relation encountered in the real world, and describe its equivalence classes and the corresponding quotient set. They must also select a real-world relation which satisfies exactly two out of the three defining properties of an equivalence relation, and explain in real-world terms why the classes of that relation do not form a partition. This assignment also involves a peer critique and feedback component.

3. Evaluation

Near the end of each semester, students were given two embargoed feedback forms. Students were required to complete these as part of the grade, and I could see who had completed the surveys; but the responses themselves were embargoed until after submission of semester grades.

3.1. Reflection on Intellectual Standards.

The first survey was a *Reflection on the Intellectual Standards*. Respondents were asked to select the two "most fundamental" of the Standards, and answer the following open-ended questions:

- Provide an example of where the standard came into play in one of your assignments/activities in the course.
- How do you predict the standard you picked will be important to you in the future?

Among these responses, the modal choice of "most fundamental" Standard was *logic* (33%), followed by *clarity* (19%) and *adaptability* (14%).

Most of the open-ended justifications for choosing logic were straightforward: propositional and predicate logic is a major component of the coursework. The other responses, however, were as a whole genuinely reflective.

> "In several of my proofs, what I wrote would be logically sound, but I could get carried away in just writing down a sequence without fully thinking about or explaining why one step leads to another. However, I realized that when I sat down and really explained my reasoning, it not only helped my proofs become clearer, but explaining also helped me understand exactly what I was doing and helped me develop a bigger picture of what is going on."

> "Almost all [progressing] proofs I received had a problem with clarity. If your arguments are confusing, or they leave a lot of grey area, they are simply not good arguments"

> "Each of our problem sets challenged us to adapt the skills we were taught in class to a novel application. This wasn't easy, but showed me the scope of what can be accomplished with adaptability."

On open-coding responses to the second question, 32% mentioned success in subsequent mathematics or math education major courses. 29% mentioned applying the Standard in daily life. 16% mentioned a specific application to their career. 13% mentioned general academic success. I reproduce a few of these responses:

> "I hope to continue to build on all of the aforementioned skills that will complement my aspirations as leader and mentor for other research scientists and engineers."

> "I think it almost needs no explanation that clarity will continue to be important in Math and education in the future. It's also just a critical life skill for college-age people to learn. I know that I am personally trying to develop positive relationships in my life, work, and school. Clear communication is critical for that."

> "This class has made me aware that I am slowly becoming more focused on a week to week sort of learning when I should be focusing on the conceptual whole of a class and how it links together to form a level of knowledge about a subject."

3.2. CCT Assessment. Students also submitted a short instrument which asked them to assess how participation in the course affected their use of CCT behaviors. They were asked to assign Likert scores from 1 (no gain) to 5 (great gain) for the following items.

- Before I begin a project, I define for myself the purpose and scope of the work.
  1: 0% 2: 0% 3: 16.7% **4: 75%** 5: 8.3%
- When I gather information, I evaluate it for accuracy, relevance and bias.
  1: 0% 2: 16.7% 3: 25% **4: 33.3%** 5: 25%
- In interpreting data or evidence, I check that my conclusions are logical and follow from the evidence.
  1: 0% 2: 0% 3: 8.3% 4: 41.7% **5: 50%**
- I ask myself whether I have considered all options when solving a problem or addressing an issue.
  1: 0% 2: 8.3% 3: 16.7% 4: 16.7% **5: 58.3%**

- I take time to think through the implications and consequences of selecting one alternative over another.
  1: 0% 2: 0% 3: 25% 4: 33.3% **5: 41.7%**
- In considering a problem or issue, I question my assumptions.
  1: 0% 2: 0% 3: 25% 4: 33.3% **5: 41.7%**
- In considering a problem or an issue, I study or consider points of view that conflict with my own.
  1: 0% 2: 8.3% **3: 33.3%** **4: 33.3%** 5: 25%

The above items do not use the language of CCT as discussed in the course, though they are clearly about important CCT behaviors. From these results it is clear that students felt participation in the course had a strong positive impact on their incorporation of CCT behaviors, with perhaps the exception of the last item regarding conflicting points of view.

The following two items asked about absolute use of the Standards, rather than improvement relative to the beginning of the semester, and were rated on a scale of 1 (never) to 5 (almost always):

- In preparing work for a class, I use relevant intellectual standards.
  1: 0% 2: 0% 3: 8.3% **4: 58.3%** 5: 33.3%
- In making decisions in my everyday life (e.g. major purchases, voting, deciding how to spend my time), I use relevant intellectual standards.
  1: 0% 2: 0% 3: 33.3% 4: 8.3% **5: 58.3%**

4. Challenges for the Instructor

The most challenging aspect of this project was simply implementing it all consistently. However, consistency is vitally important. Students in a transitions course typically undergo somewhat of a culture shock anyway; asking them to adopt an explicitly reflective stance about their own mathematical work is met in the first few weeks with something like bare disbelief. *Do I really have to do the reflection journal? Thats weird!* is what I imagine many of them saying; and the most important thing to do is to keep sending the message that yes, these tasks are important and useful.

Luckily, the specifications grading system saves an instructor a lot of time — while one might agonize over whether to give a proof 6 or 7 points out of 10, its almost instantaneous to identify proofs that are clear, complete, and correct; proofs that have no hope of rescue; and proofs that can be fixed with a little effort. Instead of using ones marking efforts to explain or justify to a student where they lost points, the instructor now is free to give tips, ask pointed questions, etc. This ends up taking less time, and its less psychologically taxing (especially when dealing with proofs that exhibit deep but correctable flaws) for me. It also helps the students start to view instructor feedback as a positive thing — red ink helps them improve their work — rather than associating it with a suffering grade.

Another place to spend this freed-up time is responding to the reflection journals. Because I implemented my reflection journals in Google Drive, it was not much effort to set aside a time each week to go through and leave comments. When students know their reflections are being read and responded to, they really get the message that the reflection matters. This is especially important in the early weeks. In future I have set a goal to respond to each students reflection journal each week. Even when I fell short of this, I had more than one student who used Google Drive's *resolve comment* feature, which notified me every time that they had thought about my response.

5. Future work

Apart from the specific content of thetwo creative assignments, nearly all of the CCT structure could be adapted to other course content. An institution which uses linear algebra or real analysis as its first proofs course, for example, would have a relatively straightforward time adapting this framework to fit its students' needs. It would be quite interesting to implement a CCT program in a calculus or college algebra course, though each would present unique challenges.

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